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Shallow-water edge waves above an inclined bottom slowly varied in along-shore direction

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Abstract

The dynamics of the shallow-water linear edge waves above an inclined bottom slowly varied in an along-shore direction is studied. By using the asymptotic method, the offshore structure of the edge waves and their dispersion relation are determined in the leading order, and the wave amplitude – in next order. The asymptotic theory confirms the "energetic" approach that the wave amplitude can be derived from the energy flux conservation in the leading order. Three different offshore bottom profiles are considered: the beach of constant slope, exponential shelf, and step-shelf. The variations of the wave amplitude and along-shore wave number are calculated in details for the cases when the parameters of the shelf zone are changed slowly in the along-shore direction.

1. Introduction

The edge waves play an important role in the dynamics of the coastal zone. Such waves have been detected in the wave field in the oceanic coastal zone many times; see for instance [1–4]. Edge waves are often considered as the major factor of the long-term evolution of the irregular coastal line, forming the rhythmic crescentic bars [5–7]. The book by Komar [6] gives several excellent pictures of coastal line structures induced by the edge waves. Laboratory experiments and rough estimations of characteristic scales are in good agreement with real observation of the coastal morphology features. But the energetics of such processes has not been investigated. Fredsoe and Deigaard [8] point out that up to now no quantitative models describing beach evolution under the edge wave action have been developed.

Short-scale edge waves are usually generated by incident wind waves due to strong nonlinearity of the wind wave field [9–13]. Large-scale edge waves are an important component in the sea disturbances produced by cyclones moving along coastlines [14]. The existing of the trapped waves explains the non-uniform character of the tsunami heights along coastline [15–17]. Totally, approximately 70% of the tsunami wave energy propagates along the Kurile Islands in Pacific as the trapped waves [18].

The modal structure of the linear barotropic trapped waves in a basin of variable depth is described in papers [19–23] and books [24,25]. Recently Constantin [26] obtained some exact solutions of the nonlinear governing equations describing a family of explicit rotational waves above a beach of constant slope. Mathematically this task is very difficult taking into account the two-dimensional character of the eigenvalue problem (vertical and offshore coordinates). Only in the shallow water approximation the eigenvalue problem contains one coordinate (offshore coordinate), and the variable depth plays the role of the potential function in the corresponding Sturm–Liouville equation. In this case the properties of the edge waves can be

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investigated in many details for arbitrary bottom profiles [21,22,24,25]. Shallow-water edge waves as energetic waves are very often observed in the coastal zone, and as a result, the shallow water approximation is popular in geophysical applications. The nonlinear theory of long edge and shelf waves is also developed: the Korteweg–de Vries equation is derived for shelf waves [27], the nonlinear correction for the phase speed of the edge waves is calculated [28–30] and the nonlinear Schrödinger equation for the wave envelope is derived [31–33], the nonlinear interactions of the edge wave triad are analysed [34–36]. We would like to mention also the weakly dispersive model for edge waves developed by Sheremet and Guza [37].

The majority of theoretical studies consider the basin with cylindrical geometry when the water depth is a function of the offshore coordinate only. The real situation is more complicated as the two-dimensional variability of the water depth has to be taken into account. Stoker and Johnson [38] analysed the trapping and scattering of the topographic waves by estuaries and headlands. Recently Baquerizo et al. [39] considered the edge wave scattering by permeable coastal structures, which are perpendicular to the coastline. Chen and Guza [40,41] analysed the resonant scattering of progressive edge waves by alongshore-periodic topography. Their results suggest that naturally occurring rhythmic features such as beach cusps and crescentic bars possess sometimes the amplitude large enough to scatter a significant amount of incident low-mode edge wave energy. Here we neglect the alongshore periodicity of the topography, but take into account the slow variation of the bottom slope in the alongshore direction. The geometry considered is that the water depth is a function of both horizontal coordinates with the strong dependence from offshore coordinate, x, and weak dependence from the along-shore coordinate, y. The asymptotic procedure using slowly varied wave parameters in the along-shore direction is described in Section 2. The results of analytical and numerical calculations of the evolution of the edge wave in the various basins are given in Section 3. These results can be used for selection of zones with large energy of the edge waves.

2. Slowly varied edge waves

In linear theory, the shallow-water edge waves can be found from the equation for the water displacement, η,

$$\frac{\partial^2\eta}{\partial t^2} - g\,\mathrm{div}\big[h(x,y)\nabla\eta\big] = 0, \tag{1}$$

where $h(x,y)$ is the basin depth, g is gravity acceleration, and the differential operators div and grad act only in the horizontal plane (x,y). Corresponding boundary conditions for the solutions of Eq. (1) are:

on the shoreline, $x_0(y)$, determined by $h(x_0(y),y)\equiv 0$, wave field should be bounded providing no penetration to the land,
far from the shoreline $(x\to\infty)$, the wave field vanishes.

We will analyse the monochromatic waves, described by $\exp(\mathrm{i}\omega t)$, and reduce (1) to

$$\mathrm{div}\big[h(x,y)\nabla\eta\big] + \frac{\omega^2}{g}\eta = 0. \tag{2}$$

In the standard approach when the depth is a function of the offshore coordinate, x only, the variables in (2) are separated,

$$\eta(x,y) = F(x)\exp(-\mathrm{i}ky), \tag{3}$$

and $F(x)$ and k are satisfied to the eigenvalue problem

$$\widehat{\Gamma}F = \frac{\mathrm{d}}{\mathrm{d}x}\left[h(x)\frac{\mathrm{d}F}{\mathrm{d}x}\right] + \left(\frac{\omega^2}{g} - h(x)k^2\right)F = 0 \tag{4}$$

with the corresponding boundary conditions on the shoreline $(x = x_0)$ and far from the beach $(x\to\infty)$. The structure of the eigenfunctions depends on the bottom relief and the wave frequency. For instance, for the beach of constant slope, $h=\alpha x$, the edge waves are called the Stokes edge waves; they are

$$\eta_n(x,y,t) = A_n\exp(-k_nx)L_n(2k_nx)\exp\big(\mathrm{i}(\omega t - k_ny)\big), \tag{5}$$

$$\omega = \sqrt{(2n+1)\alpha g k_n}, \tag{6}$$

where L_n is Laguerre polynomial and A_n is the wave amplitude. For the fixed wave frequency there are infinity modes of the Stokes waves. Their offshore structure and dispersion relation are demonstrated in Fig. 1 (for beach slope, 10^{-3}).

Now we consider the generalization of the bottom topography when the depth is presented in the form, $h = h(x,\varepsilon y)$, where $\varepsilon \ll 1$ is a small parameter. Due to the smoothness of the bottom topography in the alongshore direction the wave should be in the leading order in the form (3) with slowly variable parameters in the alongshore direction [40],

$$\eta(x,y) = A(Y)F(x,Y)\exp(-\mathrm{i}\theta) + \varepsilon\eta_1(x,Y)\exp(-\mathrm{i}\theta) + \varepsilon^2\cdots, \tag{7}$$

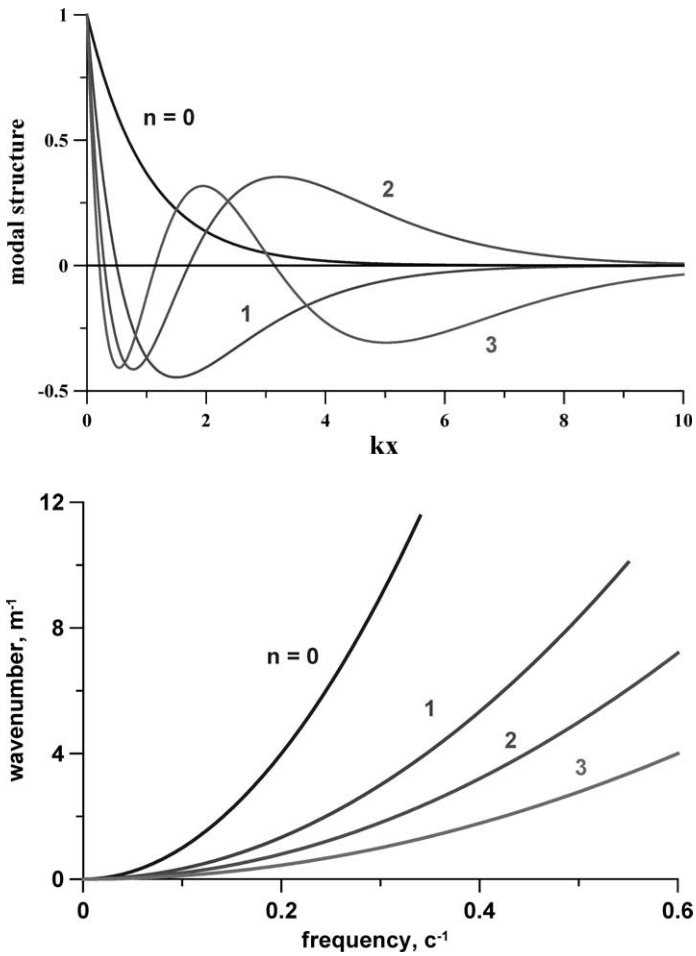

Fig. 1. Offshore structure and dispersion relation for the Stokes edge waves (slope, 10^{-3}).

where $Y = \varepsilon y$, $\theta(y)$ is a phase, and

$$k(Y) = \frac{\mathrm{d}\theta}{\mathrm{d}y} \tag{8}$$

is an alongshore wave number. We also assume that the modal function, $F(x, Y)$ is normalized on its maximum.

After the substitution of (7) and (8) in (2) and selection of the terms at the same order of the small parameter ε we obtain in the leading order (ε^0) the eigenvalue problem (4) in the offshore direction for the fixed value of the along-shore coordinate, Y. Therefore the function, F and alongshore wave number, k may be found in this order. The wave amplitude, $A(Y)$ is not determined in the leading order.

In the first order, ε^1, we have the inhomogeneous eigenvalue problem

$$\widehat{\Gamma}\eta_1 = H_1, \tag{9}$$

where

$$H_1 = 2\mathrm{i}kh\frac{\partial}{\partial Y}(AF) + \mathrm{i}AF\frac{\partial}{\partial Y}(kh). \tag{10}$$

The inhomogeneous eigenvalue problem, as it is known, is solvable if the right-side function, H_1 in (9) is orthogonal to the eigenfunctions of the adjoint operator to Γ. Using the boundary conditions, $h(x_0, Y) = 0$ for the coastline, and $\eta(x \to \infty) \to 0$

far from the coast, it is easily to show that the operator Γ is self-adjoint. In this case to solve Eq. (9) it is enough to use the orthogonality to the eigenfunction F,

$$\int\limits_{x_0(Y)}^{\infty} H(x,Y)F(x,Y)\,\mathrm{d}x = 0. \tag{11}$$

This is a well-known condition of the compatibility of the inhomogeneous self-adjoint eigenvalue problems.

After some simple mathematical manipulations Eq. (11) is reduced to

$$\int\limits_{x_0(Y)}^{\infty} \frac{\partial}{\partial Y}\bigl[k(Y)h(x,Y)A^2(Y)F^2(x,Y)\bigr]\,\mathrm{d}x = 0. \tag{12}$$

Taking into account that for coastline $h(x_0(Y),Y) = 0$ Eq. (12) is equivalent to the conservation of the integral

$$A^2(Y)k(Y)\int\limits_{x_0(Y)}^{\infty} h(x,Y)F^2(x,Y)\,\mathrm{d}x = \mathit{const}. \tag{13}$$

Another equation, for $k(Y)$ is found from the eigenvalue equation (4) in the leading order which can be also re-written in the integral form,

$$k^2(Y)\int\limits_{x_0(Y)}^{\infty} h(x,Y)F^2(x,Y)\,\mathrm{d}x = \frac{\omega^2}{g}\int\limits_{x_0(Y)}^{\infty} F^2(x,Y)\,\mathrm{d}x - \int\limits_{x_0(Y)}^{\infty} h(x,Y)\left(\frac{\partial F(x,Y)}{\partial x}\right)^2\mathrm{d}x. \tag{14}$$

As a result the wave amplitude, $A(Y)$, wave number, $k(Y)$ and the offshore modal structure, $F(x,Y)$ are determined and can be used for investigating the edge wave evolution in the basin with a slowly varied bottom and coastline in the alongshore direction.

As usually in the asymptotic methods the results of the first order can be interpreted in the framework of the average (on time) energy flux conservation, which is

$$S = \iint pu\,\mathrm{d}x\,\mathrm{d}z = \mathit{const}. \tag{15}$$

Here p is the pressure, and u is the along-shore component of the velocity. Taking into account that in shallow-water approximation the pressure is hydrostatic,

$$p = \rho g(\eta - z), \tag{16}$$

the along-shore velocity in the progressive wave is

$$u(x,y,t) = \frac{gk}{\omega}\eta(x,y,t), \tag{17}$$

after the integration on the wave period, Eq. (15) coincides with (13). So, the asymptotic method confirms the applicability of the energetic approach to describe the wave parameters in the leading order.

3. Examples of the edge wave transformation

Let us consider several examples of the edge wave evolution in the basin with different bottom topography.

3.1. Plane beach

The first example is the Stokes edge waves along the inclined bottom with slowly variable slope, $\alpha(Y)$. The modal structure of the Stokes edge waves is described by the expression (5), it is a function of $k(x - x_0(Y))$ only. The wave number of the Stokes edge wave can be found from the dispersion relation (6), and therefore,

$$k \sim \frac{1}{\alpha}. \tag{18}$$

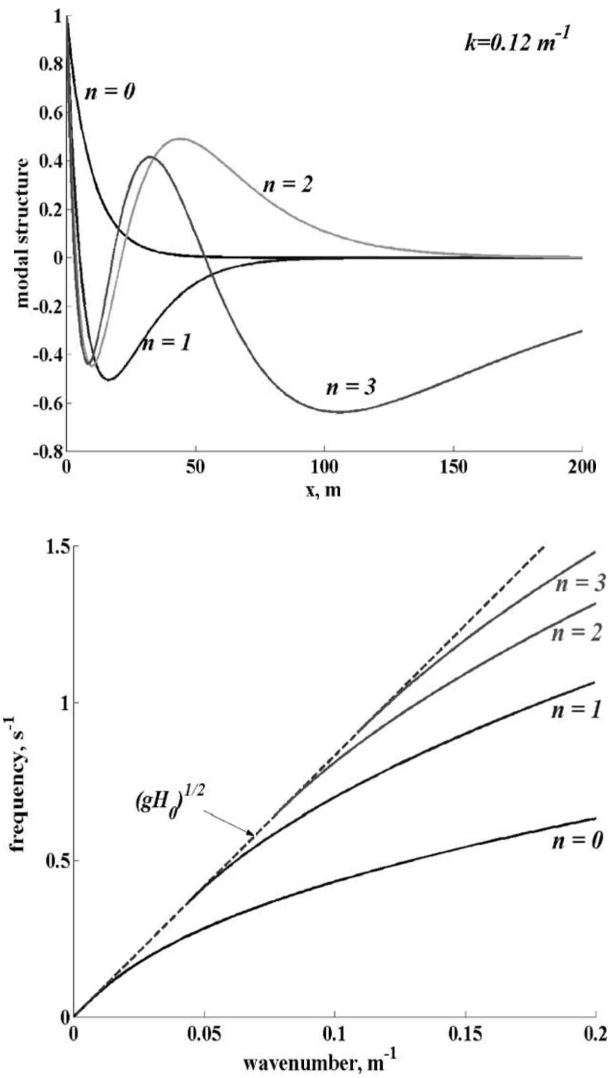

Fig. 2. Offshore structure and dispersion relation of edge waves above the exponential shelf.

After the substitution of (5) in (13) it will be found for any mode of the Stokes edge waves that

$$A \sim \frac{1}{\alpha}. \tag{19}$$

It is important to mention that wave parameters depend on the bottom slope and not on the curvature of the coastline. If the bottom slope decreases the wave amplitude growths and its wavelength (in the along-shore direction) decrease too. The characteristic offshore length of the modal function also decreases. As a result the wave became steepest and more localized in the near-shore zone.

3.2. Exponential shelf

The next bottom profile has two independent parameters: constant depth on infinity, H_0 and characteristic width of the shelf, a^{-1}

$$h(x) = H_0\big(1 - \exp(-ax)\big). \tag{20}$$

Ball [19] was the first to investigate the eigenvalue problem (4) for this bottom profile (see also [25]). The dispersion relation and eigenfunctions can be found explicitly

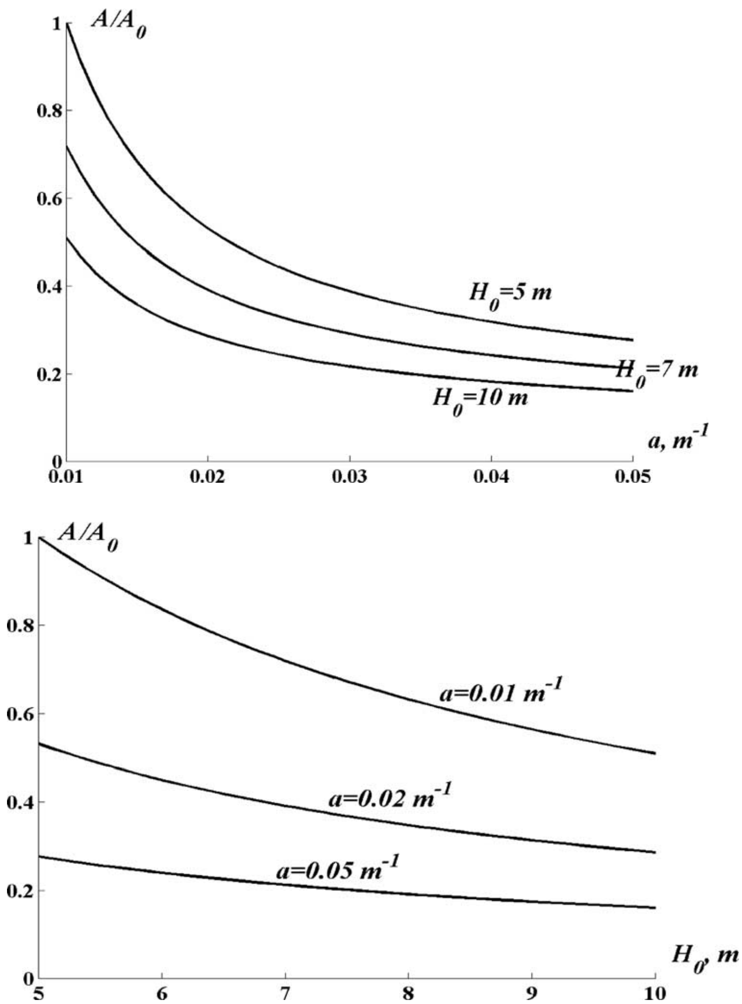

Fig. 3. Wave amplitude variation of the first mode of edge waves on exponential shelf.

$$\omega^2 = \frac{gH_0a^2}{2}\left((2n+1)\sqrt{1+\frac{4k_n^2}{a^2}} - \left(2n^2+2n+1\right)\right), \tag{21}$$

$$F_n(k,x) = \exp(-apx)\frac{\Gamma(2p+1)}{\Gamma(2p+n+1)}\sum_{j=0}^{n}\frac{n!}{j!(n-j)!}\frac{\Gamma(2p+n+j+1)}{\Gamma(2p+j+1)}(-1)^j\exp(-ajx), \tag{22}$$

where $\Gamma(z)$ is the gamma-function, and

$$p = \frac{1}{2}\left(\sqrt{1+\frac{4k^2}{a^2}} - (2n+1)\right). \tag{23}$$

The dispersion relation (21) for the shelf parameters, $H_0 = 7$ m and $a = 3.14 \times 10^{-2}$ m^{-1} and the offshore structure of the eigenfunction (22) for $k = 0.12$ m^{-1} are given in Fig. 2. These values of the shelf parameters characterize the Slapton Beach, South Devon where one of the first observations of the edge waves were made [1].

It is important to mention that the modes of the edge waves have the cut-off frequencies

$$\omega_n^{\min} = a\left[gH_0n(n+1)\right]^{1/2}, \tag{24}$$

and dispersion curves have long-wave asymptotics with the speed determined by the deep-water speed (it is shown by the dash line in Fig. 2).

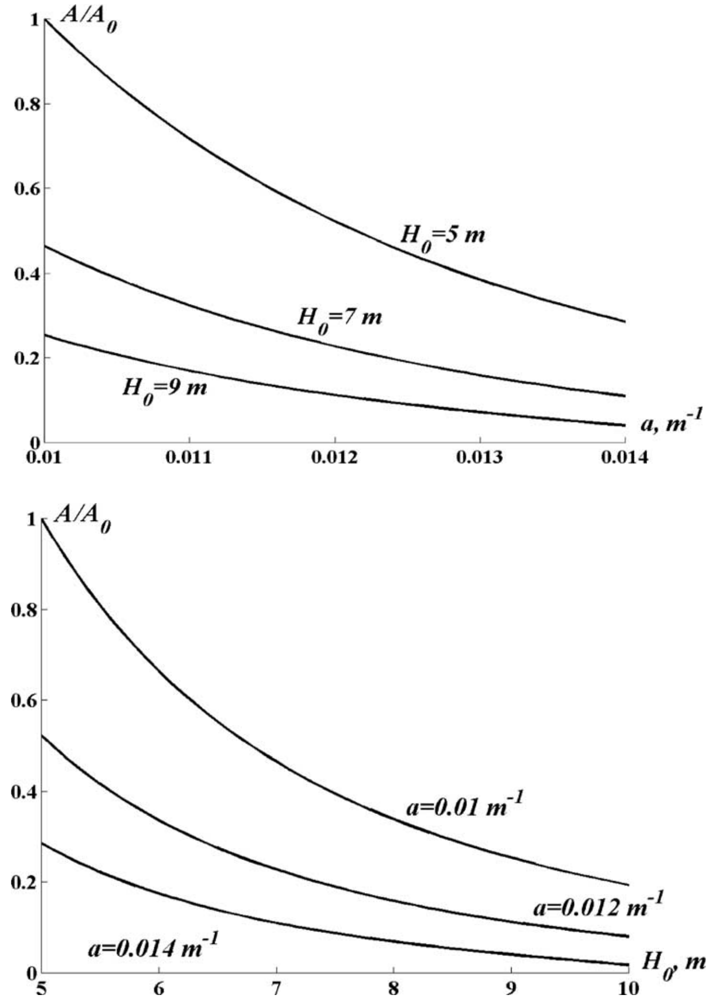

Fig. 4. Wave amplitude variation of the second mode of edge waves on exponential shelf.

If the parameters of the exponential shelf (H_0, a) are varied slowly in the along-shore direction the wave number is also varied according to (21). The variation of the wave amplitude can be found from (13) after the substitution (22). The integral (13) is calculated explicitly. For $n = 0$ it is

$$A^2 \sim \frac{1}{H_0\sqrt{(2\omega^2 + gH_0a^2)^2 - g^2H_0^2a^4(1 - 2\omega^2/(2\omega^2 + gH_0a^2))}}. \tag{25}$$

Fig. 3 illustrates the variation of the wave amplitude of the first mode ($n = 0$) when the shelf parameters are changed. Wave amplitude is normalized on A_0, calculated from (25) for $H_0 = 5$ m and $a = 0.01$ m^{-1}. When the depth, H_0 increases or the shelf width, a^{-1} decreases the wave amplitude decreases too. In fact, parameter, a is proportional to the bottom slope on the shore, and its decrease results in the wave increasing, as it was obtained for the Stokes edge waves.

The similar calculations are made for highest modes with the use of MATLAB5.3. For instance, for the second mode ($n = 1$) the variation of the wave amplitude is described by the expression

$$A^2 \sim \frac{q(1+q)(1+2q)(3+2q)}{H_0\Theta\sqrt{(2\omega^2/(gH_0a^2)+5)^2 - 9}}, \tag{26}$$

where

$$\Theta = 4q^3 + 12q^2 + 11q + 3 - 2c^2q\left(2q^2 + 3q + 1\right) + cq(c+2)\left(4q^2 + 8q + 3\right) - 2q(2c+1)\left(2q^2 + 5q + 3\right), \tag{27}$$

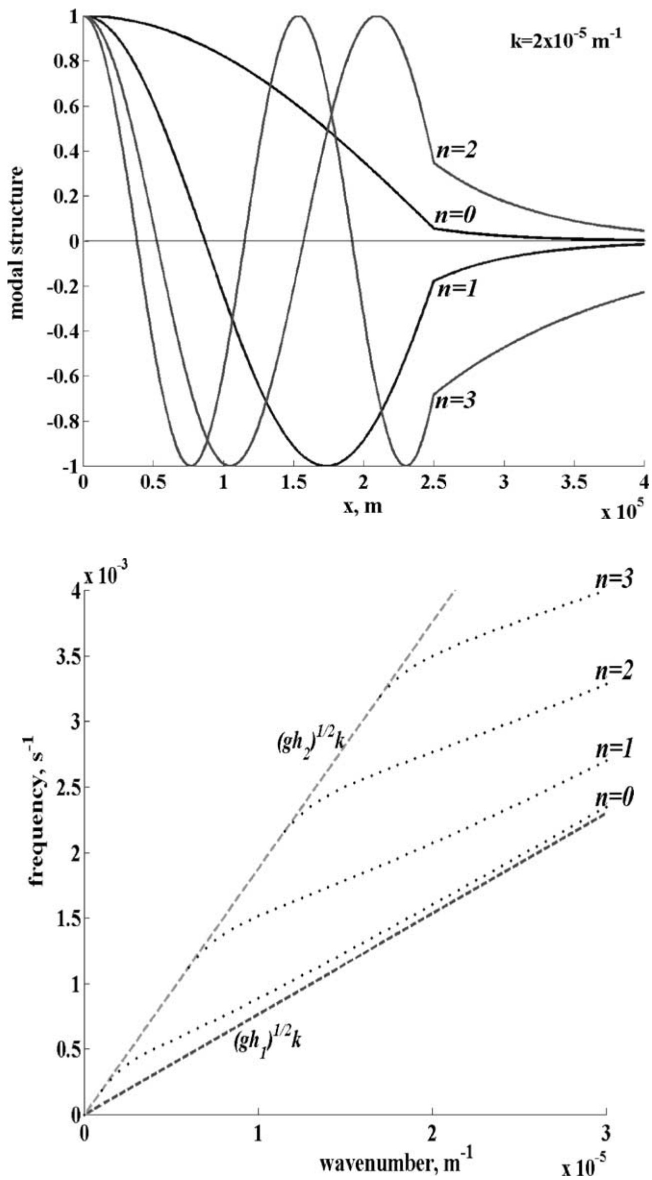

Fig. 5. Offshore structure and dispersion relation of edge waves above the step-shelf.

$$c = \frac{\Gamma(2q+3)\Gamma(2q+1)}{\Gamma^2(2q+2)}, \qquad q = \frac{1}{3}\left(\frac{\omega^2}{gH_0a^2} - 2\right).$$

Variation of the wave amplitude of the second mode of the edge waves above exponential shelf is shown in Fig. 4. Again wave amplitude decreases when the shelf became deeper and narrower.

3.3. Step-shelf

The last example is the flat shelf is described by

$$h(x) = \begin{cases} h_1, & 0 \leqslant x < l, \\ h_2, & x \geqslant l, \end{cases} \qquad h_2 > h_1. \tag{28}$$

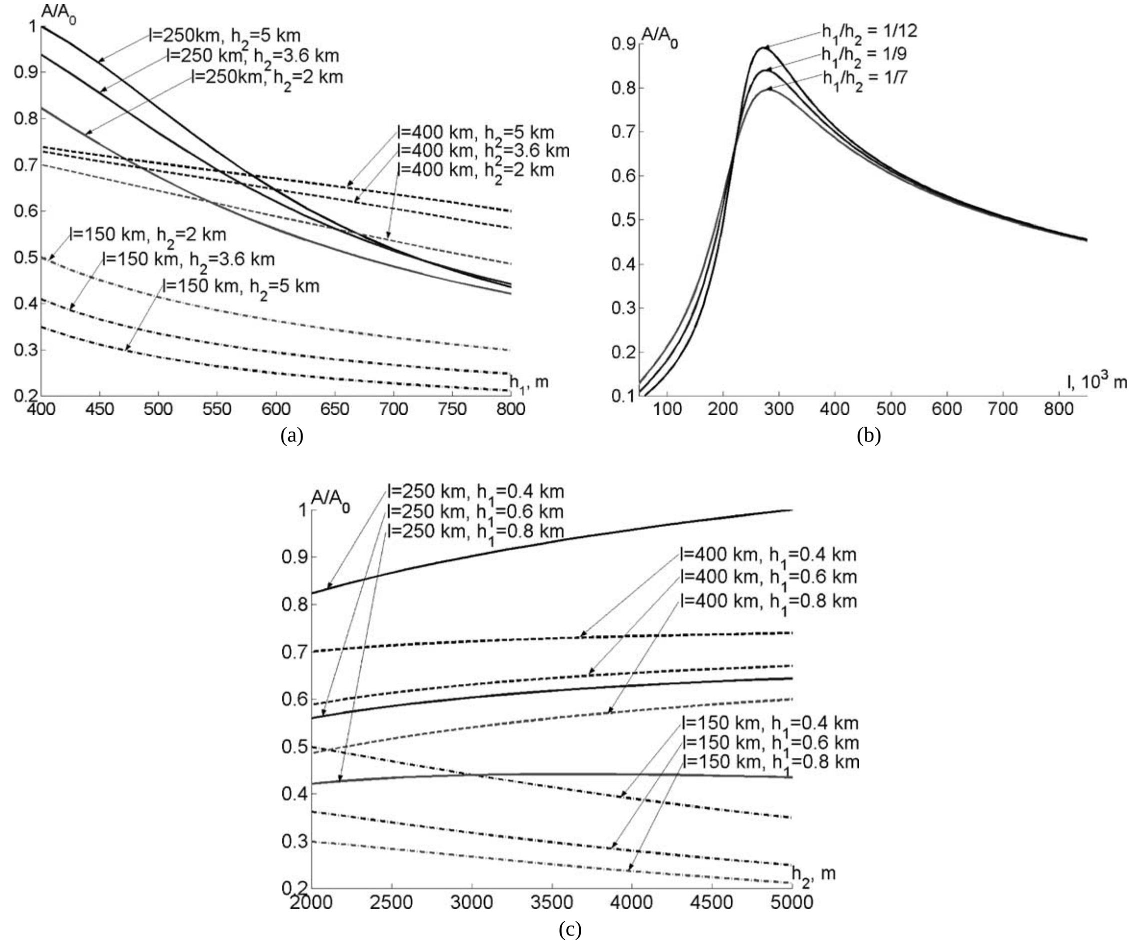

Fig. 6. Wave amplitude variability: (a) for variable shallow-water depth, (b) for variable shelf width, (c) for variable deep-water depth.

The edge waves in the basin of this geometry are studied in many papers; see, for instance, [22,25,40]. The offshore structure of the edge waves is described in the elementary functions

$$F_1(k,x) = \cos\frac{\mu x}{l}, \quad 0 \leqslant x < l, \qquad F_2(k,x) = \cos\mu \exp\left(-\sqrt{k^2 - \frac{\omega^2}{gh_2}}(x-l)\right), \quad x \geqslant l, \tag{29}$$

where

$$\mu^2 = \frac{\omega^2 l^2}{gh_1} - k^2 l^2, \tag{30}$$

and the wave number is varied in limits

$$\frac{\omega^2}{gh_2} < k^2 < \frac{\omega^2}{gh_1}. \tag{31}$$

The dispersion relation is given by the transcendental equation

$$\mu \tan\mu = \frac{h_2}{h_1}\sqrt{k^2 l^2 - \frac{\omega^2 l^2}{gh_2}} \quad \text{or} \quad \sqrt{\frac{\omega^2}{gh_1} - k^2}\tan\left(l\sqrt{\frac{\omega^2}{gh_1} - k^2}\right) = \frac{h_2}{h_1}\sqrt{k^2 - \frac{\omega^2}{gh_2}}. \tag{32}$$

The offshore structure of the edge waves and the dispersion relation are shown in Fig. 5 for the characteristic parameters of the California's shelf ($h_1 = 0.6$ km, $h_2 = 3.6$ km, $l = 250$ km) used in the paper by Snodgrass et al. [42].

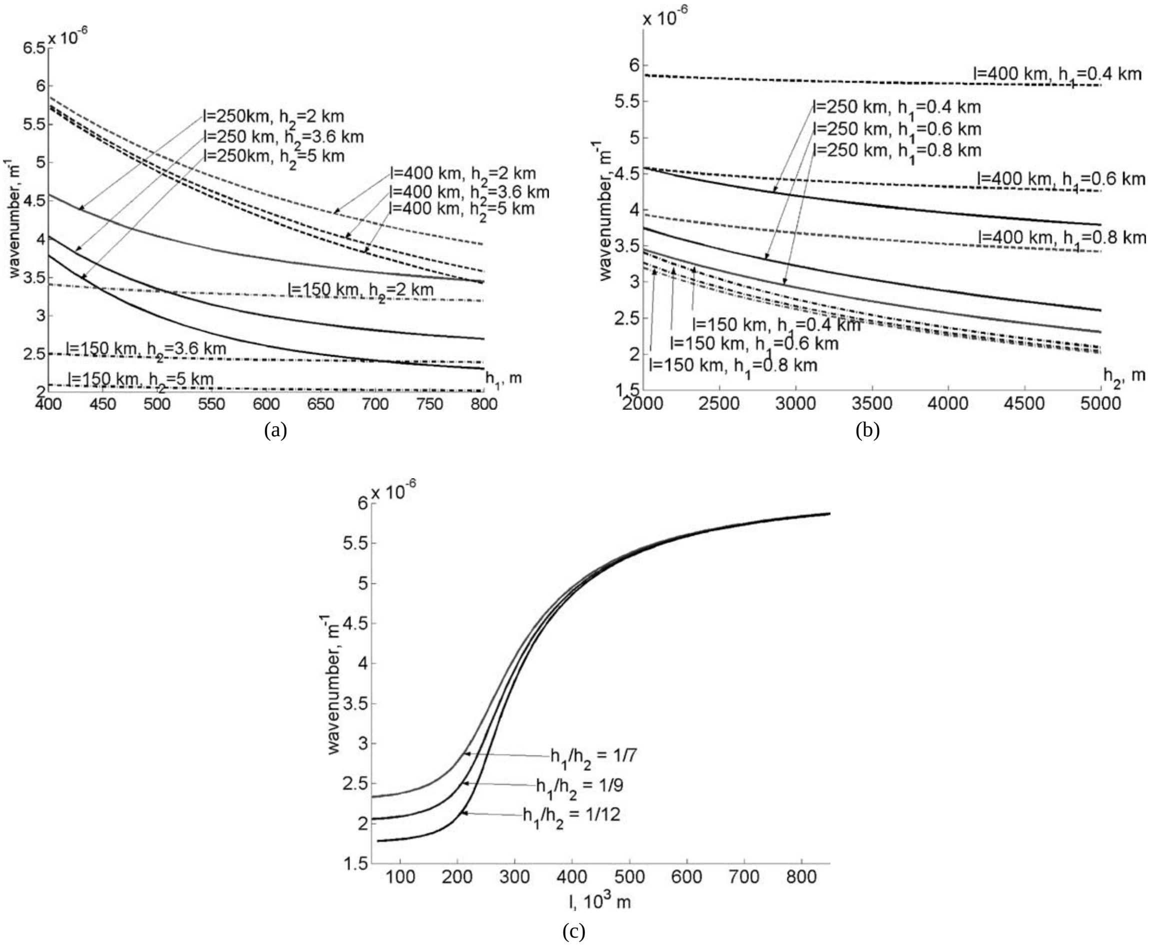

Fig. 7. Variation of the along-shore wave number: (a) for variable shallow-water depth, (b) for variable deep-water depth, (c) for variable shelf width.

The integral (13) describing the alongshore variation of the edge waves above the step-shelf can be calculated explicitly for any mode, it is

$$A^2 \sim \frac{1}{k\{h_1 l + h_1 \sin 2l\sqrt{\omega^2/(gh_1) - k^2}/(2\sqrt{\omega^2/(gh_1) - k^2}) + h_2 \cos^2 l\sqrt{\omega^2/(gh_1) - k^2}/\sqrt{k^2 - \omega^2/(gh_2)}\}}. \tag{33}$$

Fig. 6 demonstrates the variability of the wave amplitude of the first mode when the parameters of the flat shelf vary slowly. The wave amplitude is normalized on A_0 calculated for $h_1 = 0.4$ km, $h_2 = 5$ km, $l = 250$ km. When the shallow-water depth increases the wave amplitude also decreases (Fig. 6(a)). This is in quality the agreement with the calculations for other models of the shelf zone. When the width of the shelf varies the wave amplitude is varying non-monotonically due to the shelf resonance (Fig. 6(b)). As a result the dependence from the deep-water depth is also non-monotonic (Fig. 6(c)).

Fig. 7 demonstrates the variation of the along-shore wave number according to (32). For fixed wave frequency the wave number decreases when the depth increases as for the basin of constant depth; see (a) and (b). When the shelf width increases the wave number increases also (Fig. 7(c)).

4. Conclusion

The adiabatic theory of the shallow-water linear edge waves above the inclined bottom slowly varying in the along-shore direction is developed. By using the asymptotic method the amplitude evolution is found. It is shown that the asymptotic method confirms the applicability of the energy flux conservation to calculate wave amplitude in the leading order. Three different geometries of bottom topography are studied: the beach of constant slope (in offshore direction), exponential shelf, and step-shelf. The variation of the wave amplitude and along-shore wave number is calculated when the parameters of the shelf zone are changed slowly in, by the along-shore direction. Slowly varied bathymetry can amplify the edge waves, and calculated values of the amplification factors (coefficients of wave transformation) can be used to select zones of large-amplitude edge waves. Due to dispersion the edge waves may focus in space and time [43]. Both factors can lead to the appearance of the anomalous high edge waves in the coastal zone.

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